SIMULTANEOUS DETERMINATION OF THERMOPHYSICAL CHARACTERISTICS OF MATERIALS

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Nonstationary methods for the simultaneous experimental determination of the diffusivity and thermal conductivity of a semi-infinite body and a system of cylinders are analyzed.

Most nonstationary methods of determining the diffusivity and thermal conductivity use analytic solutions of the heat-conduction equation for homogeneous bodies of simple geometrical shapes. The most frequently employed initial condition is a uniform temperature distribution. The choice of boundary conditions depends on the possibility of realizing them and the desire to obtain the simplest form of analytic solution. Therefore experiments are ordinarily performed with a semi-infinite rod for which analytic solutions can be obtained for certain boundary conditions. The simplest case is the maintenance of a constant temperature during the whole time of the experiment:

$$T(x=0,\ \tau \geqslant 0) = T_c = \text{const.} \tag{1}$$

In this case the solution has the form

$$\theta(x,\ \tau) = \frac{T(x,\ \tau) - T_0}{T_c - T_0} = \operatorname{erfc} \frac{x}{2\sqrt{a\tau}}.$$

The diffusivity a can be determined by measuring the time τ and the excess temperature at an arbitrary point of the rod $x = R$ and monitoring the constancy of the excess temperature $T_c - T_0$. In this case, however, it is impossible to determine the second important characteristic of the material – the thermal conductivity λ.

Basically there are two ways to determine a and λ simultaneously for a body of this same geometric form: 1) by introducing a boundary condition which describes heat transfer at the heated end of the rod; 2) by satisfying the infinity condition by using a rod with known thermal characteristics as a standard.

In the first case one of the boundary conditions (2) or (3) is chosen:

$$\lambda \frac{\partial T(0,\ \tau)}{\partial x} = q_c = \text{const.} \tag{2}$$

In this case

$$T(x,\ \tau) - T_0 = \frac{2q_c}{\lambda}\sqrt{a\tau}\ \operatorname{ierfc} \frac{x}{2\sqrt{a\tau}}.$$

By measuring the excess temperatures in the plane $x = R$ at times τ' and τ'' we find a from the relation

$$\frac{T(R,\ \tau'') - T_0}{T_c - T_0} = \sqrt{\frac{\tau''}{\tau'}}\ \frac{\operatorname{ierfc} R/2\sqrt{a\tau''}}{\operatorname{ierfc} R/2\sqrt{a\tau'}},$$

and by measuring the constant heat flux q_c we can calculate λ. For the boundary condition

$$\lambda \frac{\partial T(0,\ \tau)}{\partial x} + \alpha [T_c - T(0,\ \tau)] = 0, \tag{3}$$

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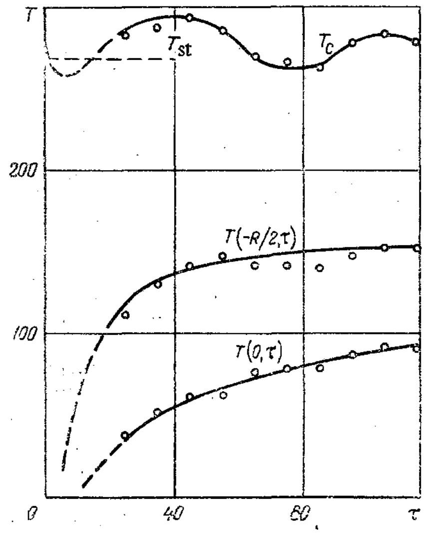

Fig. 1. Temperature T of sample as a function of the time τ in minutes [4].

the solution takes the form [1]

$$\theta(x,\ \tau)=\frac{T(x,\ \tau)-T_0}{T_c-T_0}=\operatorname{erfc}\frac{x}{2\sqrt{a\tau}}-\exp\left(\frac{\alpha x}{\lambda}+\frac{\alpha^2 a\tau}{\lambda^2}\right)\operatorname{erfc}\left(\frac{x}{2\sqrt{a\tau}}+\frac{\alpha}{\lambda}\sqrt{a\tau}\right).$$

Here also it is sufficient to measure two values of the temperature difference between the body and the medium $T(R,\ \tau)-T_0$ at times τ' and τ'' and to monitor the constancy of the temperature T_c. Thus two equations are obtained with arguments a and λ which can be solved graphically. The correct determination of the heat-transfer coefficient α is of vital importance here. In obtaining the solution it is assumed that α is constant but, strictly speaking, this is not observed in nonstationary processes.

Since boundary condition (1) is easier to maintain accurately than the others, experiments with a standard in principle give a better result and simultaneously ensure a determination of both coefficients. This method was used in [4] with a very simple experimental arrangement. The solution of the differential equation for the test body, a finite rod with thermal insulation and moisture-proofing, has the form

$$\theta(x,\ \tau)=\frac{T_1(x,\ \tau)-T_0}{T_c-T_0}=\operatorname{erfc}\frac{x}{2\sqrt{a_1\tau}}-\sum_{n=1}^{\infty}h^n\left[\operatorname{erfc}\frac{2nR-x}{2\sqrt{a_1\tau}}-\operatorname{erfc}\frac{2nR+x}{2\sqrt{a_1\tau}}\right],\quad 0\leqslant x\leqslant R,\quad \tau>0. \tag{4}$$

From a measurement of $\theta_1(R/2,\ \tau)$, or better still $\theta_1(R/3,\ \tau)$, it is possible to find a_1 by neglecting all terms of the sum even if $|h|$ is not very much smaller than 1. This is sufficient to ensure that $\theta_1(R/3,\ \tau)$ ~ 0.55 since

$$0.55=\operatorname{erfc}\frac{R/3}{2\sqrt{a_1\tau}}-1\left[\operatorname{erfc}\frac{5R/3}{2\sqrt{a_1\tau}}-\operatorname{erfc}\frac{7R/3}{2\sqrt{a_1\tau}}\right]$$

$$=\operatorname{erfc}0.422-1\,[\operatorname{erfc}2.110-\operatorname{erfc}2.954]=0.55064-0.00282.$$

In neglecting the sum we make an error of less than 0.5%. After determining a it is necessary to take account of the first term in the sum and to determine λ, since

$$h=\frac{1-k_\varepsilon}{1+k_\varepsilon};\quad k_\varepsilon=\frac{\lambda_1}{\lambda_2}\sqrt{\frac{a_2}{a_1}}=\frac{1-h}{1+h}. \tag{5}$$

The first term of the sum can be included in the calculation by increasing the time of the experiment so long as it does not become very long, or by measuring another temperature, for example at $x = R$. It may be better to combine these two methods. Thus we obtain

$$\theta_1(R,\ \tau')=(1-h)\operatorname{erfc}\frac{R}{2\sqrt{a_1\tau'}}+h\operatorname{erfc}\frac{3R}{2\sqrt{a_1\tau'}}.$$

The last term on the right hand side can frequently be neglected. For example when $\tau' = 2.25\tau$ and $R/3 \cdot 2\sqrt{a_1\tau}\sim 0.4$ the error is

$$\delta,\ \%\approx 0.15h/(1-h).$$

Thus the solution with the boundary condition q_c = const at $x = 0$ can be used even without measuring the heat flux [3].

The methods described for determining λ and a are characterized by the simplicity of the experimental arrangement and the measurement procedure. Only the temperature and time need to be measured very accurately. Therefore it is recommended that these methods be used whenever possible, i.e., for good and average heat conductors.

For very good insulators there is no simple way to ensure sufficient thermal insulation of the lateral surface of a semi-infinite rod. This is particularly difficult in investigating moist bodies when the thermal insulation must also serve as moisture-proofing and its thickness is limited by the load limit of the scales ordinarily used in the experiment.

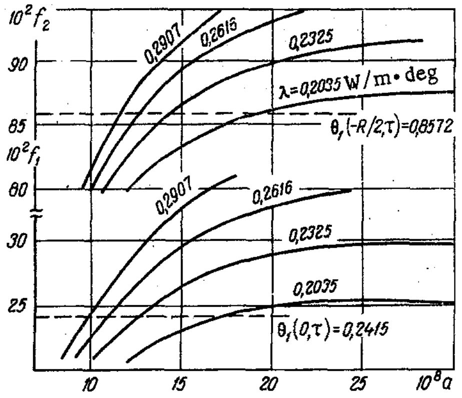

Fig. 2. The functions $f_1(a)$ and $f_2(a)$; a is in m^2/sec.

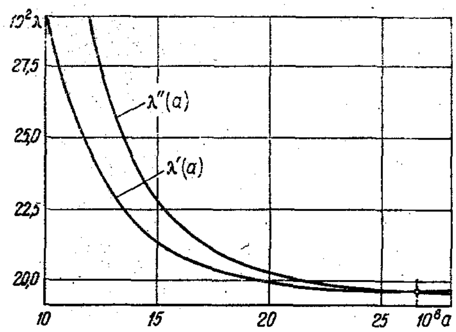

Fig. 3. The functions $\lambda'(a)$ and $\lambda''(a)$; a is in m^2/sec.

The arrangement described in [4] can be used to determine the thermophysical coefficients of insulators, but then it is impossible to avoid an inconvenient boundary condition including the heat-transfer coefficient α. For compact solid materials the sample must be a rather long cylinder of very small diameter and without lateral insulation. Then it is accurate enough to reduce the two-dimensional problem to one dimension:

$$\rho c \frac{\partial T(x, r, \tau)}{\partial \tau} = \lambda \left[\frac{\partial^2 T(x, r, \tau)}{\partial x^2} + \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T(x, r, \tau)}{\partial r} \right) \right], \tag{6}$$

$$0 \leqslant x \leqslant \infty, \quad 0 \leqslant r \leqslant r_0, \quad \tau > 0,$$

$$T(x, r, 0) = T_0 = \text{const}, \tag{7}$$

$$T(0, r, \tau) = T_c = \text{const}, \tag{8}$$

$$T(\infty, r, \tau) = T_0; \ \partial T(\infty, r, \tau)/\partial x = 0, \tag{9}$$

$$-\lambda \frac{\partial T(x, r_0, \tau)}{\partial r} = \alpha [T(x, r_0, \tau) - T_0]. \tag{10}$$

For a small radius r_0 we have $T(x, r, \tau) \approx T(x, r_0, \tau)$ and the radially symmetric temperature distribution is sufficiently accurately given by

$$T(x, r, \tau) = T(x, 0, \tau)[1 - B(r/r_0)^2].$$

Then Eq. (6) becomes

$$\rho c \frac{\partial T(x, r_0, \tau)}{\partial \tau} = \lambda \left[\frac{\partial^2 T(x, r_0, \tau)}{\partial x^2} + \frac{2}{r_0} \frac{\partial T(x, r_0, \tau)}{\partial r} \right]. \tag{6'}$$

Taking account of boundary condition (10) and omitting r_0 as an argument of T we reduce (6') to the form

$$\frac{\partial T(x, \tau)}{\partial \tau} = a \frac{\partial^2 T(x, \tau)}{\partial x^2} - \frac{2\alpha}{\rho c r_0} [T(x, \tau) - T_0]. \tag{6''}$$

Thus for very thin cylinders the two-dimensional Eq. (6) reduces to (6") with the boundary conditions (7), (8), and (9). Its solution is

$$\theta(x, \tau) = \frac{T(x, \tau) - T_0}{T_c - T_0} = \frac{1}{2} \left[\exp\left(-x \sqrt{\frac{2\alpha}{\lambda r_0}} \right) \operatorname{erfc} \left(\frac{x}{2\sqrt{a\tau}} - \sqrt{\frac{2\alpha a\tau}{\lambda r_0}} \right) + \exp\left(x \sqrt{\frac{2\alpha}{\lambda r_0}} \right) \operatorname{erfc} \left(\frac{x}{2\sqrt{a\tau}} + \sqrt{\frac{2\alpha a\tau}{\lambda r_0}} \right) \right]. \tag{11}$$

By measuring two temperatures $\theta(R/2, \tau)$ and $\theta(R, \tau)$ solution (11) becomes a system of two equations with the arguments λ and a. The graphical method for solving them is described in the following somewhat more complicated problem.

System of Thin Uninsulated Cylinders (Finite and Semi-Infinite). The values of λ and a for moist materials depend strongly on the moisture content. It is known that a temperature gradient produces a transport of moisture and therefore a uniform moisture distribution cannot be maintained. The moisture distribution will be approximately uniform in a short cylinder of moist material with the necessary surface moisture-proofing. Semiboundedness must be achieved with a dry standard. In this case the following system must be solved (sample 1, standard 2)

$$\frac{\partial T_1(x,\tau)}{\partial\tau}=a_1\frac{\partial^2T_1(x,\tau)}{\partial x^2}-\frac{2\alpha}{\rho_1c_1r_0}T_1(x,\tau)\quad(-R\leqslant x\leqslant 0),\tag{12}$$

$$\frac{\partial T_2(x,\tau)}{\partial\tau}=a_2\frac{\partial^2T_2(x,\tau)}{\partial x^2}-\frac{2\alpha}{\rho_2c_2r_0}T_2(x,\tau)\quad(0\leqslant x\leqslant \infty),\tag{13}$$

$$T_1(x,0)=T_2(x,0)=T_2(\infty,\tau)=0,\tag{14}$$

$$T_1(-R,\tau)=T_c=\text{const},\tag{15}$$

$$T_1(0,\tau)=T_2(0,\tau),\tag{16}$$

$$\frac{\partial T_1(0,\tau)}{\partial x}=\frac{\lambda_2}{\lambda_1}\frac{\partial T_2(0,\tau)}{\partial x}.\tag{17}$$

The solution can be obtained very simply by using the following form of the Laplace transform:

$$L[f(\tau)]\equiv\int_0^\infty\exp[-(s-2\alpha/\rho c r_0)\tau]f(\tau)\,d\tau.$$

For example (12) has the form

$$\left(s-\frac{2\alpha}{\rho_1c_1r_0}\right)T_{L_1}\left(x,s-\frac{2\alpha}{\rho_1c_1r_0}\right)=a_1T''_{L_1}\left(x,s-\frac{2\alpha}{\rho_1c_1r_0}\right)-\frac{2\alpha}{\rho_1c_1r_0}T_{L_1}\left(x,s-\frac{2\alpha}{\rho_1c_1r_0}\right).$$

If temperatures are measured from T_0 instead of from 0 the solutions have the form

$$\theta_1(x,\tau)=\frac{2[T_1(x,\tau)-T_0]}{T_c-T_0}=\sum_{n=0}^{\infty}h^n\Bigg\{\exp\left[-(2nR+R+x)\sqrt{\frac{2\alpha}{\lambda_1r_0}}\right]\operatorname{erfc}\left(\frac{2nR+R+x}{2\sqrt{a_1\tau}}-\sqrt{\frac{2\alpha a_1\tau}{\lambda_1r_0}}\right)+\exp\left[(2nR+R+x)\sqrt{\frac{2\alpha}{\lambda_1r_0}}\right]\operatorname{erfc}\left(\frac{2nR+R+x}{2\sqrt{a_1\tau}}+\sqrt{\frac{2a_1\tau\alpha}{\lambda_1r_0}}\right)-h\left[\exp\left[-(2nR+R-x)\sqrt{\frac{2\alpha}{\lambda_1r_0}}\right]\operatorname{erfc}\left(\frac{2nR+R-x}{2\sqrt{a_1\tau}}-\sqrt{\frac{2a_1\tau\alpha}{\lambda_1r_0}}\right)+\exp\left[(2nR+R-x)\sqrt{\frac{2\alpha}{\lambda_1r_0}}\right]\operatorname{erfc}\left(\frac{2nR+R-x}{2\sqrt{a_1\tau}}-\sqrt{\frac{2\alpha a_1\tau}{\lambda_1r_0}}\right)\right]\Bigg\},\tag{18}$$

$$\theta_2(x,\tau)=\frac{2[T_2(x,\tau)-T_0]}{T_c-T_0}=(1-h)\sum_{n=0}^{\infty}h^n\Bigg\{\exp\left[-\left(\frac{2nR+R}{\sqrt{a_1}}+\frac{x}{\sqrt{a_2}}\right)\sqrt{\frac{2\alpha a_1}{\lambda_1r_0}}\right]\operatorname{erfc}\left(\frac{2nR+R}{2\sqrt{a_1\tau}}+\frac{x}{2\sqrt{a_2\tau}}-\sqrt{\frac{2\alpha a_1\tau}{\lambda_1r_0}}\right)\cdot\exp\left[\left(\frac{2nR+R}{\sqrt{a_1}}+\frac{x}{\sqrt{a_2}}\right)\sqrt{\frac{2\alpha a_1}{\lambda_1r_0}}\right]\operatorname{erfc}\left(\frac{2nR+R}{2\sqrt{a_1\tau}}+\frac{x}{2\sqrt{a_2\tau}}+\sqrt{\frac{2\alpha a_1\tau}{\lambda_1r_0}}\right)\Bigg\}.\tag{19}$$

By measuring two temperatures $\theta_1(-R/2,\tau)$ and $\theta_1(0,\tau)=\theta_2(0,\tau)$ and the time τ we obtain from (18) two equations with arguments λ_1 and a_1. We illustrate the graphical determination of λ_1 and a_1 (henceforth written λ and a) by an example.

In the experiment described in [4] the best quality No. 2 cut tobacco was investigated. In experiment No. 3 tobacco with a 17.3% moisture content and a density of 350 kg/m³ was used. The standard was paraffin with $\lambda_2=0.267$ W/m·deg, $c_2=3.22\cdot10^3$ J/kg·deg, and $\rho_2=910$ kg/m³. The insulation was plastic with $\lambda_i\sim0.21$. The measured temperatures are shown in Fig. 1 as a function of the time.

Using the one-dimensional solution (4) and the experimental values it was calculated that $\lambda=0.0352$ W/m·deg and $a=6.97\cdot10^{-8}$ m²/sec. This result shows that plastic cannot serve as a thermal insulator; in fact there is almost no thermal insulation for bodies with $\lambda=0.035$. However, the value of λ obtained is a consequence of the crude calculational error, and the accurate calculation using Eq. (4) for x = R gives

$$h = 1 - \frac{\theta_1(0.025;\ 1500)}{\operatorname{erfc} 0.025/2\sqrt{6.97\cdot 10^{-8}\cdot 1500}} = 1 - \frac{0.123}{0.084} = -0{,}464,$$

$$\lambda_1 = \frac{1-h}{1+h}\lambda_2\sqrt{\frac{a_1}{a_2}} = 0.6175.$$

Even this result, of course, cannot be correct since:

$$c_1 = \lambda_1/\rho_1 a_1 = 0.6175\cdot 10^8/350\cdot 6.97 = 25.3\cdot 10^3 \ \text{J/kg}\cdot\text{deg}.$$

Thus instead of (4) we should use (18) with $x = -R/2$ and $x = 0$ for $\tau = 1500$ sec, set up two equations with arguments $a(a_1)$ and $\lambda(=\lambda_1)$ and find the point of intersection of the corresponding curves. It is assumed that $R = 0.025$ m and $r_0 = 0.0125$ m; instead of α the heat transfer coefficient k must be calculated since the rod is insulated by plastic with $d = 0.06$ m:

$$k = 1\Big/\left(\frac{1}{\alpha} + \frac{d}{2\lambda_i}\ln\frac{d}{d_0}\right) \approx 4{,}36 \ \text{W/m}^2\cdot\text{deg}.$$

For a horizontal cylinder in a free air jet

$$\alpha = 8.2 + 0.00733\,(\Delta T)^{1.33} \quad \text{kcal/m}^2\cdot\text{deg}\cdot\text{h} \quad \text{(Schack)},$$

$$\alpha = 8.1 + 0.045\,\Delta T \quad \text{kcal/m}^2\cdot\text{deg}\cdot\text{h} \quad \text{(Cammeter)}.$$

Using these data we obtain the following functions from (18):

$$f_1(a,\ \lambda) = \theta_1(0;\ 1500;\ a,\ \lambda); \quad f_2(a,\ \lambda) = \theta_1(-0.0125;\ 1500;\ a,\ \lambda).$$

We calculate a sufficient number of points by computer for the curves $f_1(a)$ and $f_2(a)$ with the parameter λ and draw them in the fa plane (Fig. 2). From the intersection of the curves for $f_1(a)$ and $\theta_1(0,\ 1500)$ = const we obtain several points for a new curve $\lambda = \lambda''(a)$. In the same way we obtain the curve $\lambda = \lambda''(a)$. These curves are drawn in the λa plane (Fig. 3), and their intersection gives the values of the required quantities a and λ.

It is clear from Fig. 1 that the galvanometer readings were not sufficiently stable during the experiment, and therefore it is proper to take the values of the temperatures from the curve. Thus

$$T_1(0;\ 1500) = 26.08\,^\circ\text{C}; \quad T_1 = (-0.0125;\ 1500) = 37.50\,^\circ\text{C}.$$

The temperatures $T_1(-R,\ \tau) = T_c$ = const requires particular attention. It can be seen from [4] that it is difficult to maintain the temperature accurately constant. Starting from the assumption that at the beginning of the experiment the instability was still larger the behavior of $T_1(-R,\ \tau)$ can be traced in the 0-25 min range (open curve of Fig. 1). Then the average value $T_{st} \cong 58.7$°C, and consequently

$$\theta_1(0;\ 1500) = \frac{2\,(26.08 - 21.6)}{58.7 - 21.6} = 0.2415; \ \theta_1(-0.0125;\ 1500) = 0.8572.$$

Thus from Fig. 3 we find $\lambda = 0.196$ W/m·deg, $a = 26.4\cdot 10^{-8}$ m²/sec, and consequently c = 2120 J/kg·deg. This value of c is realistic.

CONCLUSIONS

The values of λ and a can be determined simultaneously by using a rather simple experimental arrangement [4] for all sides of materials. Equation (4) must be used for good heat conductors, Eq. (11) for compact insulators, while moist insulators require the use of moisture-proofing and Eq. (18).

If experiments are performed in freely circulating air there will not be a large error in estimating α or k; the error in estimating α(k) does not have a very large effect on λ but does have a very large effect on a:

k	2,91	3,49	4,17	4,36	4,65
λ	0,2245	0,2045	0,1954	0,1960	0,2047
$a\cdot 10^8$	11,45	13,31	18,62	26,40	30,00

The temperatures must be measured very accurately, particularly at the junctions of the cylinders which are rather far from the heated surface. The temperature $T_1(-R, \tau) = T_c$ must give an immediate pulse for controlling the thermostat. Its constancy can be ensured by the condensation of water vapor.

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